Sequences Vs. SERIES

I'm a list of #'s

I'm an infinite sum

If we have a sequence: $\boxed{\{a_n\}} = \{a_1, a_2, a_3, a_4, \ldots\}$

"N^{th} partial sum of $\sum a_n$"

... we can form a new sequence, the sequence of partial sums.

$$\boxed{S_N = \sum_{n=1}^{N} \boxed{a_n}} = a_1 + a_2 + a_3 + \cdots + a_N$$

the limit of S_N gives the value of the series:

$$\lim_{N \to \infty} \boxed{S_N} = \boxed{\sum_{n=1}^{\infty} \boxed{a_n}} = a_1 + a_2 + a_3 + a_4 + \cdots\cdots$$

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Sequence $\{S_n\}$ of partial sums

$$S_1 = \sum_{n=1}^{1} \boxed{a_n} = a_1$$

$$S_2 = \sum_{n=1}^{2} a_n = a_1 + a_2$$

$$S_3 = \sum_{n=1}^{3} a_n = a_1 + a_2 + a_3$$

$$\vdots$$

$$S_N = \sum_{n=1}^{N} a_n = a_1 + a_2 + a_3 + \dots + a_N$$

$N \to \infty$

$$S = \boxed{\sum_{n=1}^{\infty} \boxed{a_n}} = a_1 + a_2 + a_3 + \dots\dots$$

Sequence $\{a_n\}$

Series $\sum a_n$

Sequences Vs. SERIΣS

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Sequence/series - land	Function - land
$\{a_n\}$	$f(x)$
$\sum_{n=1}^{\infty} a_n$	$\int_1^{\infty} f(x)\,dx$
$\{S_N\}$	$\int_1^{R} f(x)\,dx$

$$\sum_{n=1}^{\infty} a_n = \lim_{N\to\infty} S_N \qquad \int_1^{\infty} f(x)\,dx = \lim_{R\to\infty}\int_1^{R} f(x)\,dx$$

Sequences vs. SERIES

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$$\{a_n\}$$

Goal: Compute $\lim_{n\to\infty} a_n$

1. If a_n can be written as a function $f(x)$, then can use Calc 1.

   i.e., replace every "n" by "x".

2. $\lim_{n\to\infty} |a_n| = 0$
   $\Downarrow$
   $\lim_{n\to\infty} a_n = 0$

3. If a_n is monotonic and bounded, then it converges.

4. $\lim_{n\to\infty} r^n = 0$ if $|r|<1$

5. $\lim_{n\to\infty} \frac{1}{n^r} = 0$ if $r>0$

6. $\lim_{n\to\infty} \left(1+\frac{a}{n}\right)^{bn} = e^{ab}$

7. $\lim_{n\to\infty} \sqrt[n]{n} = 1$

8. limits involving (!)
   - divide and conquer
   - sometimes, just simplify.

$$\sum_{n=1}^{\infty} a_n$$

Goal: Converges or diverges (compute sum if possible).

1. Divergence Test:
   If $\lim_{n\to\infty} \boxed{a_n} \neq 0$, then $\sum_{n=1}^{\infty} \boxed{a_n}$ diverges.

2. Geometric Series:
   $$\sum_{n=1}^{\infty} ar^{n-1} = \begin{cases} \frac{a}{1-r} & |r|<1 \\ \text{diverges} & |r|\geq 1 \end{cases}$$
   (can also write $\sum_{n=0}^{\infty} ar^n$)

COMING SOON!!!

3. Telescoping series $\sum_{n=1}^{\infty} a_n$
   $a_n = b_n - b_{n+i}$ ← i is usually 1 or 2.

4. Integral Test: If $a_n = f(n)$ for a function $f(x)$ which is positive, continuous, decreasing, then $\sum_{n=1}^{\infty} a_n$ converges $\iff \int_1^{\infty} f(x)\,dx$ converges

5. P-Series
   $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p>1$, diverges if $p\leq 1$