

Sequences

I'm a list of #'s

Vs.

SERIES

I'm an infinite sum

If we have a sequence:

$$\{a_n\} = \{a_1, a_2, a_3, a_4, \dots\}$$

... we can form a new sequence, the sequence of partial sums.

"Nth partial sum of $\sum a_n$ "

$$S_N = \sum_{n=1}^N a_n = a_1 + a_2 + a_3 + \dots + a_N$$

the limit of S_N gives the value of the series:

$$\lim_{N \rightarrow \infty} S_N = \sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + a_4 + \dots$$

Sequences \swarrow Vs.

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Sequence $\{S_n\}$ of partial sums

$$S_1 = \sum_{n=1}^1 a_n = a_1$$

$$S_2 = \sum_{n=1}^2 a_n = a_1 + a_2$$

$$S_3 = \sum_{n=1}^3 a_n = a_1 + a_2 + a_3$$

...

$$S_N = \sum_{n=1}^N a_n = a_1 + a_2 + a_3 + \dots + a_N$$

↓

$N \rightarrow \infty$

$$S = \sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \dots$$

Sequence $\{a_n\}$

Series $\sum a_n$

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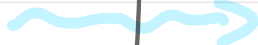
Vs.

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Sequence/series - land

Function - land

$\{a_n\}$



$f(x)$

$\sum_{n=1}^{\infty} a_n$

a_n



$\int_1^{\infty} f(x) dx$

$\{S_N\}$



$\int_1^R f(x) dx$

$$\sum_{n=1}^{\infty} a_n = \lim_{N \rightarrow \infty} S_N$$

$$\int_1^{\infty} f(x) dx = \lim_{R \rightarrow \infty} \int_1^R f(x) dx$$

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$$\{a_n\}$$

Goal: Compute $\lim_{n \rightarrow \infty} a_n$

1. If a_n can be written as a function $f(x)$, then can use Calc 1.

ie, replace every " n " by " x ".

2. $\lim_{n \rightarrow \infty} |a_n| = 0$

\downarrow
 $\lim_{n \rightarrow \infty} a_n = 0$

3. If a_n is monotonic and bounded, then it converges.

4. $\lim_{n \rightarrow \infty} r^n = 0$ if $|r| < 1$

5. $\lim_{n \rightarrow \infty} \frac{1}{n^r} = 0$ if $r > 0$

6. $\lim_{n \rightarrow \infty} \left(1 + \frac{a}{n}\right)^{bn} = e^{ab}$

7. $\lim_{n \rightarrow \infty} \sqrt[n]{n} = 1$

8. Limits involving (!)

- divide and conquer
- sometimes, just simplify.

$$\sum_{n=1}^{\infty} a_n$$

Goal: Converges or diverges (compute sum if possible).

1. Divergence Test:

If $\lim_{n \rightarrow \infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges.

2. Geometric Series:

$$\sum_{n=1}^{\infty} ar^{n-1} = \begin{cases} \frac{a}{1-r} & |r| < 1 \\ \text{diverges} & |r| \geq 1 \end{cases}$$

can also write $\sum_{n=0}^{\infty} ar^n$

3. Telescoping series $\sum_{n=1}^{\infty} a_n$
 $a_n = b_n - b_{n+1}$ ← i is usually 1 or 2.

4. Integral Test: If $a_n = f(n)$ for a function $f(x)$ which is positive, continuous, decreasing, then

$$\sum_{n=1}^{\infty} a_n \text{ converges} \iff \int_1^{\infty} f(x) dx \text{ converges}$$

5. P-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \text{ converges if } p > 1 \\ \text{diverges if } p \leq 1$$

COMING SOON!!!